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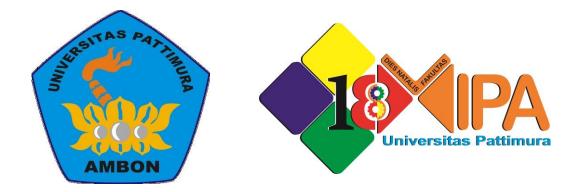
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## The 2<sup>nd</sup> International Seminar of Basic Science

"Natural Science for Exploration The Sea-Island Resources"

Poka-Ambon, 31<sup>st</sup> May 2016

Mathematic and Natural Science Faculty Universitas Pattimura Ambon 2016

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## Welcoming Address By The Organizing Committee

Today, We have to thank the The Almighty Allah SWT for the implementation of this international seminar. This is the second seminar about Basic Science in The Faculty of MIPA Pattimura University. The seminar under the title "Natural Sciences for Exploration the Sea-Island Resources" will be carried out on May 31<sup>st</sup> 2016 at Rectorate Building, Pattimura University. There are 200 participants from lecturers, research institute, students, and also there are 34 papers will be presented.

My special thanks refer to the rector of Pattimura University and the Dean of MIPA Faculty, Prof. Dr. Pieter Kakissina, S.Pd., M.Si. I also would like to express my deepest gratitude to Prof. Amanda Reichelt-Brushett, M.Sc., Ph.D. ; Kazuhiko Ishikawa, Ph.D. ; Nicolas Hubert, Ph.D. ; Prof. Dr. Kirbani Sri Brotopuspito ; Prof. Dr. Marjono, M.Phil. ; Gino V. Limon, M.Sc., Ph.D. as the keynote speakers.

The last, We hope this international seminar usefull for all of us, especially Mollucas People and very sorry if any mistake. Thank you very much.

#### Dr. La Eddy, M.Si.

Chairman of Organizing Committee

### Opening Remarks By Dean of Mathematic and Natural Sciences Faculty

I express my deepest gratitude to The Almighty God for every single blessing He provides us especially in the process of holding the seminar until publishing the proceeding of International Seminar in celebrating the 18<sup>th</sup> anniversary of MIPA Faculty, Pattimura University. The theme of the anniversary is under the title "Natural Sciences for Exploration the Sea-Island Resources". The reason of choosing this theme is that Maluku is one of five areas in Techno Park Marine in Indonesia. Furthermore, it is expected that this development can be means where the process of innovation, it is the conversion of science and technology into economic value can be worthwhile for public welfare especially coastal communities.

Having the second big variety of biological resources in the world, Indonesia is rich of its marine flora and fauna. These potential resources can be treated as high value products that demand by international market. Basic science of MIPA plays important role in developing the management of sustainable marine biological resources.

The scientific articles in this proceeding are the results of research and they are analyzed scientifically. It is expected that this proceeding can be valuable information in terms of developing science and technology for public welfare, especially people in Maluku.

My special thanks refer to all researchers and reviewers for your brilliant ideas in completing and publishing this proceeding. I also would like to express my gratefulness to the dies committee-anniversary of MIPA Faculty for your creativity and hard working in finishing this proceeding, God Bless you all.

#### Prof. Dr. Pieter Kakisina, S.Pd., M.Si.

Dean of Mathematic and Natural Sciences Faculty

## ACKNOWLEDGMENT

The following personal and organization are greatfully acknowledgment for supporting "The 2<sup>nd</sup> International Seminar of Basic Science 2016"

Hotel Mutiara Ambon

**PROCEEDINGS** The 2<sup>nd</sup> International Seminar of Basic Science May, 31<sup>st</sup> 2016

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#### THE TOTAL IRREGULARITY STRENGTH OF THE CORONA PRODUCT OF A PATH WITH A WHEEL

Faldy Tita, F. Y. Rumlawang, M. I. Tilukay, and D. L. Rahakbauw

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#### ABSTRACT

A totally irregular total *k*-labeling  $\lambda : V \cup E \rightarrow \{1, 2, ..., k\}$  of a graph *G* is a total labeling such that *G* has an edge irregular total labeling and a vertex irregular total labeling at the same time. In other words, for any two different edges (and vertices), their weights are distinct. The minimum value *k* for which *G* has a totally irregular total *k*-labeling is called the total irregularity strength of *G*, denoted by ts(G). In this paper, we determine the exact value \$ts\$ of the corona product of a path with a wheel which is equals to the lower bound.

Keywords : corona product, path, total irregularity strength, wheel

#### INTRODUCTION

Let *G* be a finite, simple, and undirected graph with vertex set *V* and edge set *E*. A total labeling of *G* is a mapping that sends  $V \cup E$  to a set of numbers (usually positive or nonnegative integers). According to the condition defined in a total labeling, there are many types of total labelings. For a total labeling  $\lambda : V \cup E \rightarrow \{1, 2, ..., k\}$ , the weight of a vertex *v* and the weight of an edge e = xy are defined by  $w(v) = \lambda(v) + \sum_{uv \in E} \lambda(uv)$  and  $w(xy) = \lambda(x) + \lambda(y) + \lambda(xy)$ , respectively. If all the vertex weights are distinct, then  $\lambda$  is called a vertex irregular total *k*-labeling. The minimum value of *k* for which there exist a vertex (or an edge) irregular total labeling  $\lambda : V \cup E \rightarrow \{1, 2, ..., k\}$  is called the total vertex (or edge) irregular total labeling  $\lambda : V \cup E \rightarrow \{1, 2, ..., k\}$  is called the total vertex (or edge) irregular total labeling  $\lambda : V \cup E \rightarrow \{1, 2, ..., k\}$  is called the total vertex (or edge) irregular total labeling  $\lambda : V \cup E \rightarrow \{1, 2, ..., k\}$  is called the total vertex (or edge) irregular total labeling  $\lambda : V \cup E \rightarrow \{1, 2, ..., k\}$  is called the total vertex (or edge) irregular total labeling  $\lambda : V \cup E \rightarrow \{1, 2, ..., k\}$  is called the total vertex (or edge) irregularity strength of *G* and is denoted by tvs(G) (tes(G)), respectively.

This notions are introduced by Baca *et al.* in [2]. They gave the boundary for the tvs(G) that for every (p,q)-graph G with minimum degree  $\delta(G)$  and maximum degree  $\Delta(G)$ ,

$$\left|\frac{p+\delta(G)}{\Delta(G)+1}\right| \le tvs(G) \le p+\Delta(G)-2\delta(G)+1;$$
(1)

and for the total edge irregularity strength of graph G, that is

$$\left|\frac{|E(G)|+2}{3}\right| \le tes(G) \le |E(G)|.$$

$$\tag{2}$$

In [8], Nurdin *et al.* determined the total edge-irregular strengths of the corona product of paths with some graphs. They found that for  $m \ge 2$  and  $n \ge 3$ ,

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$$tes(P_m \odot W_n) = \left[\frac{(3n+2)m+1}{3}\right].$$
(3)

Later, in [7], Nurdin *et al.* improved the lower bound of tvs of any connected graph *G*. For  $\delta = \delta(G)$  be the minimum degree of vertex in *G*,  $\Delta = \Delta(G)$  be the maximum degree of vertex in *G*, and  $n_i$  be the number of vertex of degree *i*, where  $\delta \leq i \leq \Delta$ ,

$$tvs(G) \ge \max\left\{ \left[ \frac{n_{\delta} + \delta}{\delta + 1} \right], \left[ \frac{n_{\delta} + n_{\delta + 1} + \delta}{\delta + 2} \right], \cdots, \left[ \frac{\sum_{i=\delta}^{\Delta} n_i + \delta}{\Delta + 1} \right] \right\}$$
(4)

For further results on tvs(G) and tes(G), one can refer to [3].

Combining previous conditions on both total labeling, Marzuki et al. in [6] introduced another irregular total labeling. A total *k*-labeling  $\lambda : V \cup E \rightarrow \{1, 2, \dots, k\}$  of *G* is called a *totally irregular total k-labeling* if for any pair of vertices *x* and *y*, their weights w(x) and w(y) are distinct and for any pair of edges  $x_1x_2$  and  $y_1y_2$ , their weights  $w(x_1x_2)$  and  $w(y_1y_2)$  are distinct. The minimum value *k* for which *G* has a totally irregular total labeling is called the *total irregularity strength* of *G*, denoted by ts(G). They gave the lower bound of ts(G),

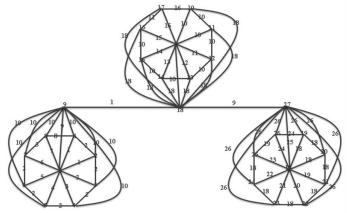
$$ts(G) \ge max\{tes(G), tvs(G)\};$$
(5)

and determined the exact value of total irregularity strength of paths and cycles. In [9], Ramdani and Salman determined the *ts* of several cartesian product graphs. Later, Ramdani et al. [11] determined the *ts* for gear graphs, fungus graphs,  $ts(Fg_n)$ , for n even,  $n \ge 6$ ; and for disjoint union of stars. Tilukay *et al.* in [12] and [13] determined the *ts* of fan, wheel, triangular book, friendship graphs, complete graph, and bipartite graph, which are equal to the lower bound. Some other results can be seen in [1], [4], [5], and [10].

In this paper, we determine the exact value *ts* of the corona product of a path with a wheel which is equals to the lower bound.

#### **RESULTS AND DISCUSSION**

The corona product of *G* with *H*, denoted by  $G \odot H$ , is a graph obtained by taking one copy of an *n*-vertex graph *G* and *n* copies  $H_1$ ,  $H_2$ ,  $\cdots$ ,  $H_n$  of H and then joining the *i*-th vertex of G to every vertex in  $H_i$ . For instance, corona product of a path  $P_3$  with a wheel  $W_8$  is given in Figure 1.



**Figure 1.**  $P_3 \odot W_8$  with totally irregular total 27-labeling

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**Theorem 2.1** For  $m \ge 2$  and  $n \ge 3$ , let  $P_m \odot W_n$ , be a corona product of a path  $P_m$  and a wheel  $W_n$  with m(n+2) vertices and (3n+2)m-1 edges. Then

$$ts(P_m \odot W_n) = \left[\frac{(3n+2)m+1}{3}\right].$$

#### Proof:

Since  $|V(P_m \odot W_n)| = m(n+2)$ ,  $|E(P_m \odot W_n)| = (3n+2)m-1$ ,  $\delta = 4$  and  $\Delta = n+2$ , by (3), (4) and (5), we have  $ts(P_m \odot W_n) \ge \left[\frac{(3n+2)m+1}{3}\right]$ Next, we proof that  $ts(P_m \odot W_n) \le \left[\frac{(3n+2)m+1}{3}\right]$ . Let  $t_i = \left[\frac{(3n+2)i+1}{3}\right]$  for  $2 \le i \le m$ . Let the

vertex set  $V(P_m \odot W_n) = \{x_i, y_i^j \mid 1 \le i \le m \text{ and } 1 \le j \le n+1\}$  and the edge set  $E(P_m \odot W_n) = \{x_i y_i^j \mid 1 \le i \le m \text{ and } 1 \le j \le n+1\} \cup \{y_i^j y_i^{j+1} \mid 1 \le i \le m \text{ and } 1 \le j \le n-1\}$ 

$$\bigcup \{y_i^1 y_i^n \mid 1 \le i \le m \text{ and } 1 \le j \le n-1\} \cup \{y_i^j y_i^{n+1} \mid 1 \le i \le m \text{ and } 1 \le j \le n\}$$

$$\int \{x_i x_{i+1} \mid 1 \le i \le m - 1\}$$

 $\cup \{x_i x_{i+1} \mid 1 \le i \le m-1\}$ From the definition above, we know that  $y_i^{n+1}$  is the center of  $W_i$ . We construct an irregular total labeling  $\lambda : V(G) \cup E(G) \rightarrow \{1, 2, 3, ..., k\}$ . We divide the construction into two cases as follows:

 $\begin{aligned} & \textbf{Case 1. } n \leq 7 \\ \lambda(x_i) = \begin{cases} 1, & \text{for } i = 1; \\ t_i - n + 1, & \text{for } 2 \leq i \leq m; \\ j, & \text{for } i = 1 \text{ and } 1 \leq j \leq n + 1; \\ \lambda(y_i^j) = \begin{cases} j, & \text{for } i = 1 \text{ and } 1 \leq j \leq n + 1; \\ t_i - n + j, & \text{for } 2 \leq i \leq m \text{ and } j = n + 1; \\ t_i, & \text{for } 2 \leq i \leq m \text{ and } j = n + 1; \end{cases} \\ \lambda(x_i y_i^j) = \begin{cases} 1, & \text{for } i = 1 \text{ and } 1 \leq j \leq n + 1; \\ (3n + 2)i - n - 1 - 2t_i, & \text{for } 2 \leq i \leq m \text{ and } 1 \leq j \leq n; \\ (3n + 2)i - n - 2t_i, & \text{for } 2 \leq i \leq m \text{ and } 1 \leq j \leq n; \\ (3n + 2)i - n - 2t_i, & \text{for } 2 \leq i \leq m \text{ and } j = n + 1; \end{cases} \\ \lambda(y_i^1 y_i^n) = \begin{cases} 3, & \text{for } i = 1; \\ (3n + 2)i + 1 - n - 2t_i, & \text{for } 2 \leq i \leq m; \\ (3n + 2)i + 1 - n - 2t_i, & \text{for } 2 \leq i \leq m; \end{cases} \\ \lambda(y_i^j y_i^{j+1}) = \begin{cases} n + 3 - j, & \text{for } i = 1 \text{ and } 1 \leq j \leq n - 1; \\ (3n + 2)i + 1 - j - 2t_i, & \text{for } 2 \leq i \leq m \text{ and } 1 \leq j \leq n - 1; \end{cases} \\ \lambda(y_i^j y_i^{n+1}) = \begin{cases} n + 2, & \text{for } i = 1 \text{ and } 1 \leq j \leq n - 1; \\ (3n + 2)i + 1 - 2t_i, & \text{for } 2 \leq i \leq m \text{ and } 1 \leq j \leq n - 1; \end{cases} \\ \lambda(x_i x_{i+1}) = \begin{cases} 4n + 2 - t_2, & \text{for } i = 1; \\ (3n + 2)i + 2n - t_i - t_{i-1}, & \text{for } 2 \leq i \leq m. \end{cases} \\ \text{Hence, we have} \end{aligned}$ Case 1.  $n \leq 7$ Hence, we have  $w(x_i y_i^j) = (3n+2)i - 3n + j,$  for  $1 \le i \le m$  and  $1 \le j \le n + 1$ ;  $w(y_i^1 y_i^n) = (3n+2)i - 2(n-1),$  for  $1 \le i \le m$ ;  $w(y_i^j y_i^{j+1}) = (3n+2)i - 2(n-1) + j,$  for  $1 \le i \le m$  and  $1 \le j \le n - 1$ ;  $w(y_i^j y_i^{n+1}) = (3n+2)i - (n-1) + j,$  for  $1 \le i \le m$  and  $1 \le j \le n;$   $w(y_i^j y_i^{n+1}) = (3n+2)i - (n-1) + j,$  for  $1 \le i \le m$  and  $1 \le j \le n;$ Hence, we have  $w(x_i x_{i+1}) = (3n+2)i + 2,$ for  $1 \leq i \leq m$ ; and  $w(x_i) = \begin{cases} n+t_2, & \text{for } i = 1; \\ n(5n+7-2t_2)+3, & \text{for } i = 2, m = 2; \\ (n+2)(3n+2)i - n^2 - n - (2n-1)t_i - t_{i-1} - t_{i+1} + 1, & \text{for } 2 \le i \le m-1, m \ne 2; \\ (n+2)(3n+2)i - n^2 - n - (2n-2)t_i - t_{i-1} + 1, & \text{for } i = m, m \ne 2; \end{cases}$ for i = 1:

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$$w(y_{1}^{j}) = \begin{cases} 2n+9, & \text{for } j = 1; \\ 3n+10-j, & \text{for } 2 \le j \le n; \\ n(n+2)+j+1, & \text{for } j = n+1; \end{cases}$$

$$w(y_{i}^{j}) = \begin{cases} 4i(3n+2)-3n-7t_{i}+2 & \text{for } 2 \le i \le m, j = 1; \\ 4i(3n+2)-2n-7t_{i}-j+3 & \text{for } 2 \le i \le m, 2 \le j \le n; \\ 4i(3n+2)-n-7t_{i}+3 & \text{for } 2 \le i \le m, j = n+1; \end{cases}$$
It easy to check that the maximum label is  $t_{i}$ . By verifying the edge weights, we have

 $\{w(x_i y_i^j)\} = \{(3n+2)i - 3n + 1, (3n+2)i - 3n + 2, \dots, (3n+2)i - 2n + 1\}; \\ \{w(y_i^1 y_i^n)\} = \{n + 4, 4n + 6, \dots, 3mn + 2m - 2n + 2\}; \\ \{w(y_i^j y_i^{j+1})\} = \{(3n+2)i - 2(n-1) + 1, (3n+2)i - 2(n-1) + 2, \dots, (3n+2)i - n + 1\}; \\ \{w(y_i^j y_i^{n+1})\} = \{(3n+2)i - (n-1) + 1, (3n+2)i - (n-1) + 2, \dots, (3n+2)i + 1\}; \\ \{w(x_i x_{i+1})\} = \{3n + 4, 6n + 6, \dots, 3mn + 2m + 2\}.$ 

Hence, the edge weights form a consecutive sequence 3, 4, ..., (3n + 2)m + 1. For the vertex weights, it can be checked that there is no two vertices of the same weight.

#### Case 2. $n \ge 8$

$$\begin{split} \lambda(x_i) &= \begin{cases} n+1, & \text{for even } n, i=1; \\ n-1, & \text{for odd } n, & i=1; \\ t_i, & \text{for } 2 \leq i \leq m; \\ \\ \lambda(y_i^j) &= \begin{cases} n-j+1, & \text{for } i=1 \text{ and } 1 \leq j \leq n; \\ 1, & \text{for } i=1 \text{ and } j=n+1; \\ t_i-n+j-1, & \text{for } 2 \leq i \leq m \text{ and } 1 \leq j \leq n+1; \\ t_i-n+j-1, & \text{for } 2 \leq i \leq m \text{ and } 1 \leq j \leq n+1; \\ 1, & \text{for even } n, & i=1 \text{ and } 1 \leq j \leq n; \\ n+1, & \text{for even } n, & i=1 \text{ and } 1 \leq j \leq n; \\ n+1, & \text{for odd } n, & i=1 \text{ and } 1 \leq j \leq n; \\ n+1, & \text{for odd } n, & i=1 \text{ and } 1 \leq j \leq n; \\ n+4, & \text{for odd } n, & i=1 \text{ and } j=n+1; \\ (3n+2)i-n-2t_i, & \text{for } 2 \leq i \leq m \text{ and } 1 \leq j \leq n+1; \\ \lambda(y_i^1y_i^n) &= ni, & \text{for } 1 \leq i \leq m; \\ \lambda(y_i^jy_i^{j+1}) &= \begin{cases} 3, & \text{for } i=1 \text{ and } j=1; \\ 2, & \text{for } i=1 \text{ and } j=n-1; \\ 1, & \text{for odd } n, & i=1 \text{ and } j=n-1; \\ 3+(i-1)n, & \text{for } 2 \leq i \leq m \text{ and } 1 \leq j \leq n-1; \\ 1+(i-1)n, & \text{for } 2 \leq i \leq m \text{ and } 1 \leq j \leq n-1; \\ 1+(i-1)n, & \text{for odd } n, 2 \leq i \leq m \text{ and } j=n-1; \\ 1, & \text{for } i=1 \text{ and } j=n-1; \\ 1, & \text{for } i=1 \text{ and } j=n-1; \\ 1, & \text{for } i=1 \text{ and } j=n-1; \\ (3n+2)i+1-j-2t_i, & \text{for } 2 \leq i \leq m \text{ and } 1 \leq j \leq n-1; \\ (3n+2)i+2-j-2t_i, & \text{for } 2 \leq i \leq m \text{ and } 1 \leq j \leq n-1; \\ (3n+2)i+2-j-2t_i, & \text{for } 2 \leq i \leq m \text{ and } 1 \leq j \leq n-1; \\ (3n+2)i+1-j-2t_i, & \text{for } 2 \leq i \leq m \text{ and } 1 \leq j \leq n-1; \\ (3n+2)i+2-j-2t_i, & \text{for } 2 \leq i \leq m \text{ and } 1 \leq j \leq n-1; \\ (3n+2)i+1-j-2t_i, & \text{for odd } n, 2 \leq i \leq m \text{ and } j=n; \\ (3n+2)i+1-j-2t_i, & \text{for odd } n, 2 \leq i \leq m \text{ and } j=n; \\ (3n+2)i+2n-t_i-t_{i+1}, & \text{for } 2 \leq i \leq m-1. \\ \\ \text{Hence.} \end{split}$$

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$$\begin{split} & w(x_i y_i^{l}) = (3n+2)i+2-j, \mbox{ for } 1 \leq i \leq m \mbox{ and } 1 \leq j \leq n+1; \\ & w(y_i^{l} y_i^{l+1}) = \begin{pmatrix} (3n+2)i-(n+1), \mbox{ for } 1 \leq i \leq m; \\ (3n+2)i-n+4, \mbox{ for } 1 \leq i \leq m \mbox{ and } j=1; \\ (3n+2)i-n+1+2j, \mbox{ for } 0d n, 1 \leq i \leq m \mbox{ and } j=n-1; \\ (3n+2)i-n+1+2j, \mbox{ for } 0d n, 1 \leq i \leq m \mbox{ and } j=n-1; \\ (3n+2)i-n+1+2j, \mbox{ for } 0d n, 1 \leq i \leq m \mbox{ and } j=n-1; \\ (3n+2)i-n+2j, \mbox{ for } 0d n, 1 \leq i \leq m \mbox{ and } j=n-1; \\ (3n+2)i-3n+3, \mbox{ for } 0d n, 1 \leq i \leq m \mbox{ and } j=n-1; \\ (3n+2)i-3n+2, \mbox{ for } ven n, 1 \leq i \leq m \mbox{ and } j=n-1; \\ (3n+2)i-3n+2, \mbox{ for } ven n, 1 \leq i \leq m \mbox{ and } j=n-1; \\ (3n+2)i-3n+1, \mbox{ for } 1 \leq i \leq m \mbox{ and } j=n; \\ w(x_i x_{i+1}) = (3n+2)i+2, \mbox{ for } 1 \leq i \leq m; \\ and \\ w(x_i) = \begin{cases} 2(n+1)+n(n+2)+1, \mbox{ for } ven n, 1 \leq i \leq m \mbox{ and } j=n; \\ 2(n+1)+n(n+4)+3, \mbox{ for } odd n, i=1; \\ (3n+2)2i+n-2t_i-t_{i+1}+2, \mbox{ for } odd n, 2 \leq i \leq m; \\ (3n+2)2i+n-2t_i-t_{i+1}+2, \mbox{ for } odd n, 2 \leq i \leq m; \\ (3n+2)2i+n-2t_i-t_{i+1}+2, \mbox{ for } odd n, 2 \leq i \leq m; \\ (3n+2)2i+n-2t_i-t_{i+1}+2, \mbox{ for } odd n, 2 \leq i \leq m; \\ 4(n+1), \mbox{ for even } n, j=1; \\ 3n-4, \mbox{ for } odd n, \mbox{ } j=1; \\ 3n-2j+7, \mbox{ for } odd n, \mbox{ } j=n-1; \\ 3n-2j+9, \mbox{ for } odd n, \mbox{ } j=n-1; \\ 2n-(-1)^n+6, \mbox{ for } odd n, \mbox{ } j=n-1; \\ 2n-(-1)^n+6, \mbox{ for } odd n, \mbox{ } j=n+1; \\ n+3+\frac{(n^2-n)}{2}, \mbox{ for } odd n, \mbox{ } j=n+1; \\ n+7+\frac{(n-2)(n+1)}{2}, \mbox{ for } odd n, \mbox{ } j=n+1; \\ (3n+2)-3n-6t_i+2, \mbox{ for } ven n, \mbox{ } 2 \leq i \leq m, \mbox{ } j=1; \\ 4i(3n+2)-3n-6t_i+3j, \mbox{ for } ven n, \mbox{ } 2 \leq i \leq m, \mbox{ } 2 \leq n-1; \\ 4i(3n+2)-3n-6t_i+4, \mbox{ for } odd n, \mbox{ } 2 \leq i \leq m, \mbox{ } 2 \leq n-1; \\ 4i(3n+2)-3n-6t_i+4, \mbox{ for } odd n, \mbox{ } 2 \leq i \leq m, \mbox{ } n=n; \\ 4i(3n+2)-3n-6t_i+4, \mbox{ for } odd n, \mbox{ } 2 \leq i \leq m, \mbox{ } n=n; \\ 4i(3n+2)-n-7t_i+3, \mbox{ for } 0dn, \mbox{ } 2 \leq i \leq m, \mbox{ } n=n; \\ 4i(3n+2)-n-7t_i+3, \mbox{ for } 0dn, \mbox$$

It easy to check that the maximum label is  $t_i$ . By verifying the edge weights, we have  $\{w(x_iy_i^j)\} = \{(3n+2)i+1, (3n+2)i, \dots, (3n+2)i+1-n\};$  $\{w(y_i^1y_i^n)\} = \{2n+1, 5n+3, \dots, 3mn+2m-n-1\};$  $\{w(y_i^jy_i^{j+1})\} = \{2n+6, 5n+8, \dots, 3mn+2m-n+4\}$  $\cup \{(3n+2)i-n+5, (3n+2)i-n+7, \dots, (3n+2)i+n-3\}$  $\cup \{(3n+2)i+n-2\} \cup \{(3n+2)i+n-1\};$  $\{w(y_i^jy_i^{n+1})\} = \{3\} \cup \{4\} \cup \{5\}$  $\cup \{(3n+2)i-n-2, (3n+2)i-n-4, \dots, (3n+2)i-3n-4\};$ 

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 $\{w(x_i x_{i+1})\} = \{3n + 4, 6n + 6, \dots, 3mn + 2m + 2\}.$ 

Hence, the edge weights form the consecutive sequence 3, 4, ..., (3n + 2)m + 1. For the vertex weights, it can be checked that there is no two vertices of the same weight. Then, we can conclude that  $\lambda$  is a totally irregular total  $t_i$ -labeling of  $P_i \odot W_n$ , for  $1 \le i \le m$ . Thus,

$$ts(P_m \odot W_n) \le \left[\frac{(3n+2)m+1}{3}\right]$$

#### **OPEN PROBLEM :**

- 1. Find  $ts(P_1 \odot W_n)$  for any (n + 1) wheel graph.
- 2. Find the total irregularity strength of the corona product of path with any graph.

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