# STUDENT ENROLLMENT ALLOCATION INTO ACADEMIC DEPARTMENTS USING WEIGHTED GOAL PROGRAMMING 

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#### Abstract

A weighted non-preemptive multi-criteria model is built to optimize the distribution of students into academic departments of a faculty by taking into account the limits of space capacity, financial allocation, the number of instructors and affirmative action quotas. Each constraint has a weight attached. This model is applied to the Faculty of Science and Technology, Universiti Kebangsaan Malaysia. The successful application demonstrates the ability of the weighted non-preemptive model to comply with the student intake requirement and constraints of academic departments in the faculty.


Keywords: Affirmative action; Allocation; Constraints; Goal; Priority;
Weighted mean

## 1. INTRODUCTION

Goal programming has been used extensively in many areas such as in management for Malaysian crops [1-4], portfolio of Malaysian stock market [5], management of tourism activities [6], library acquisition and funding allocation [7-8], food product distribution [9] and bakery production [10]. Currently goal programming models are being applied in minimization of energy consumption on multiprocessor platforms [11], fuzzy investment decisions [12], flood flow model [13], and joint decision making of inventory [14].

Earlier modeling approaches in institutions of higher learning tend to be directed towards aggregate planning of human, financial, and physical resources in the higher levels of academic administration planning [15-19]. However, the main academic thrusts of the institutions are left out. Some departmental level modeling techniques dealing with faculty-course assignment required the development of complex utility functions to express faculty preferences for certain courses [20-23]. The required time consuming modeling efforts, and the complexity and the time necessary to develop utility functions of faculty preferences could however limit their application when used on a practical reoccurring basis on departmental level in an organization of higher education [24].

In order to emphasize the thrust of academic institutions, academic administrators have to determine the number of students to be enrolled based on the expertise of academic staff, student capacity of each program, admission policies and create a racial balance in each program based on the affirmative action policy to be dealt with every semester. Administrators' decisions should indicate the thrust of the academic faculty, limited infrastructure, and the affirmative action requirement for government funded public universities.

In this paper, a weighted non-preemptive goal programming model is developed which will optimize the allocation of students into academic departments taking into account the expertise of academic staff, student capacity of each program, admission policies and financial allocations. It is further refined to create a racial balance in each program based on the affirmative action policy and provide a fair distribution of student-to-faculty ratio. Weights will be used to apportion the students into academic departments in the faculty that will reflect the research thrust of the faculty. The weighted deviations are then included in the objective function to emphasize the ranking of goals.

## 2. MODEL DEVELOPMENT

Listed below are the input parameters, constraints and the objective function of the model in allocating students of a faculty, the Faculty of Science and Technology, to its five academic departments of Bioscience and Biotechnology (BB), Physical Science (PS), Chemical Science (CS), Mathematical Science (MS), and Environmental Science (ES), for its three years undergraduate study.

## Input Parameters

$c_{j}=$ capacity of first year students in department $j ;$
$r_{j}=$ student to faculty ratio required for department $j ;$
$q_{j}=$ minimum ratio of native students over the total students entering department $j ;$
$z_{j}=$ number of drop-out native students from department $j ;$
$t_{j}=$ total capacity of students in department $j$ proportionate to the number of classes;
$e_{j}=$ number of students enrolling into year two;
$h_{j}=$ number of students enrolling into year three.

## Variables

$x_{j}=$ number of natives admitted into department $j$;
$y_{j}=$ number of non-natives admitted into department $j$;
$a_{j}=$ total number of first years in department $j ;$
$d_{j}=$ total number of students enrolled in department $j ;$
$f_{i}=$ total number of students in department $i$;
$l_{j}=$ number of faculty required for department $j ;$
$\mathrm{B}=$ faculty budget;
$J_{j}=$ budget for department $j$;
$J=$ sum of departmental budget;
$X=$ total number of first year native students admitted into the faculty;
$Y=$ total number of non-native students admitted into the faculty;
$A=$ total number of first year students admitted into the faculty.

## Constraints

The constraints involved in the Faculty of Science and Technology with five departments are as follows.

$$
\begin{equation*}
\sum_{j=1}^{M} x_{j}=X_{i}, \sum_{j=1}^{M} y_{j}=Y_{i}, \quad \sum_{j=1}^{M} a_{j}=A_{i}, \tag{1}
\end{equation*}
$$

where $M=5, X_{i}=765, Y_{i}=465$ and $A_{i}=1230$.
For $j=1, \ldots, 5$, we have

$$
\begin{align*}
& a_{j}-x_{j}-y_{j}=0,  \tag{2}\\
& a_{j}+d_{1 j}^{-}-d_{1 j}^{+}=c_{j}, \tag{3}
\end{align*}
$$

where $c_{1,}, c_{2}, c_{13}, c_{4}$, and $c_{5}$ are $260,210,260,230$ and 300 .

$$
\begin{equation*}
d_{j}+d_{2 j}{ }^{-}-d_{2 j}{ }^{+}=t_{j}, \tag{4}
\end{equation*}
$$

where $t_{1}, t_{2}, t_{3}, t_{4}$ and $t_{5}$ are $740,640,760,650$ and 900.

$$
\begin{equation*}
x_{j}-z_{j}-q_{j} a_{j}+d_{3 j}^{-}-d_{3 j}^{+}=0, \tag{5}
\end{equation*}
$$

where $q_{1}, q_{2}, q_{3}, q_{4}$ and $q_{5}$ are $0.75,0.66,0.51,0.60$ and 0.59 .

$$
\begin{equation*}
d_{j}-a_{j}=e_{j}+h_{j} \tag{6}
\end{equation*}
$$

where $e_{j}+h_{j}$ are 476, 427, 497, 427 and 602.

$$
\begin{align*}
& f=\sum_{j=1}^{5} d_{j}=3659 .  \tag{7}\\
& r_{j} \cdot l_{j}-d_{j}+d_{4 j}{ }^{-}-d_{4 j}{ }^{+}=0, \tag{8}
\end{align*}
$$

where $r_{1}, r_{2}, r_{3}, r_{4}$ and $r_{5}$ are $17,18,17,15$ and 12.

$$
\begin{equation*}
l-\sum_{j=1}^{5} l_{j}=0 . \tag{9}
\end{equation*}
$$

For the budget estimation in department $j$, we note that the students cost in a department varies with the number of students enrolled in that department. A piecewise linear relationship represents the students cost as a function of the number of students.

However, if the linear segment has been determined and the cost per student is averaged, then the budget estimation can be simplified as follows.

$$
J_{j}=\alpha_{j} d_{j} \text { and } \sum_{j=1}^{5} J_{j}-B+d_{5}^{-}-d_{5}^{+}=0
$$

where $\alpha_{1}, \alpha_{2}, \alpha_{3}, \alpha_{4}$ and $\alpha_{5}$ are $397,1298,666,140$ and 966.

## Objective Function

The criterion of optimization aims at maximizing the allocation of students accepted into the department by

$$
\begin{align*}
& \max \sum_{j=1}^{M} a_{j}=\sum_{j=1}^{M} x_{j}+y_{j} \text { first year admission }  \tag{10}\\
& \max \quad f=\sum_{j=1}^{M} d_{j}  \tag{11}\\
& \text { enrolees in department }  \tag{12}\\
& \min \quad \sum_{j=1}^{M} x_{j}-q_{j} a_{j}  \tag{13}\\
& \text { affirmative action quota }  \tag{14}\\
& \min \quad \sum_{j=1}^{M} l_{j} \\
& \min \quad \sum_{j=1}^{M} J_{j} \\
& \text { number of faculty } \\
& \text { budget constraints }
\end{align*}
$$

Note that the objective function in this case, has to be rewritten as a single function of deviations and prioritized accordingly.

Minimize

$$
\begin{align*}
Z= & \sum_{j=1}^{M} k_{1 j}\left(d_{1 j}^{-}+d_{1 j}^{+}\right)+\sum_{j=1}^{M} k_{2 j}\left(d_{2 j}^{-}+d_{2 j}^{+}\right)+\sum_{j=1}^{M} k_{3 j}\left(d_{3 j}^{-}+d_{3 j}^{+}\right)+\sum_{j=1}^{M} k_{4 j}\left(d_{4 j}^{-}+d_{4 j}^{+}\right)  \tag{15}\\
& +\left(k_{51} d_{5}^{-}+k_{52} d_{5}^{+}\right) .
\end{align*}
$$

Note that the weights $k_{i j}$ have values $1,2,3,4$ or 5 .

$$
\begin{align*}
& k_{11}=5, k_{12}=4, k_{13}=2, k_{14}=3, k_{15}=1, k_{21}=3, k_{22}=2, k_{23}=1, k_{24}=5, k_{25}=4, \\
& k_{31}=1, k_{32}=3, k_{33}=5, k_{34}=4, k_{35}=2, k_{41}=5, k_{42}=4, k_{43}=3, k_{44}=2, k_{45}=1, \\
& k_{51}=1, k_{52}=1 . \tag{16}
\end{align*}
$$

For budget expenditure, the values of $k_{51}=k_{52}=1$ implies that under expenditure and over expenditure of budget spending $\mathrm{d}_{5}{ }^{-}$and $\mathrm{d}_{5}{ }^{+}$are equally restrained with the same weightage.

## 3. ANALYSIS OF RESULTS

The output obtained with regard to the enrolment into five departments in the Faculty of Science and Technology is shown in Table 1.

Table 1. Results of the Non-Preemptive Model.

| Departments | BB | PS | CS | MS | ES |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Number of first year native students | 197 | 125 | 133 | 134 | 176 |
| Number of first year non-native students | 63 | 64 | 127 | 89 | 122 |
| Number of first year students to be admitted | 260 | 189 | 260 | 223 | 298 |
| Number of academic staff in each department | 43 | 34 | 45 | 43 | 75 |
| Number of students in each department | 736 | 616 | 751 | 649 | 894 |
| Budget allocation to each department | 292192 | 799568 | 504162 | 91000 | 869400 |

The table suggests the admission mix of students in the five academic centres. As an example, the second column of the table indicates a mix of 197 bumiputra students and 63 non bumiputra students to fill up the 260 admission capacity for the centre BB. The current figure has a mix of 194 bumiputra and 63 non bumiputra students, which is 3 students less than the 260 number of admission capacity. The fourth and fifth row enables us to calculate the students-staff ratio.

The values of the deviational variables with their respective weights are listed below. Note that the objective value is 278082.9

$$
\begin{aligned}
& \text { Weight }=5 ; \quad d_{11^{-}}=0, d_{24^{-}}=0, d_{33^{-}}=0, d_{33^{+}}=0.4, \quad d_{41^{-}}=5, d_{41^{+}}=0 . \\
& \text { Weight }=4 ; \quad d_{12^{-}}=21, d_{25^{-}}=0, d_{34^{-}}=0, d_{34^{+}}=0.2, \quad d_{42^{-}}=4, d_{42^{+}}=0 .
\end{aligned}
$$

```
Weight \(=3 ; \quad d_{14^{-}}=7, d_{21}{ }^{-}=4, d_{32^{-}}=0, d_{32^{+}}=0.26, d_{43^{-}}=0, d_{43^{+}}=8\).
Weight \(=2 ; \quad d_{13}{ }^{-}=0, d_{22^{-}}=24, d_{35^{-}}=0, d_{35^{+}}=0.18, d_{44^{-}}=5, d_{44^{+}}=0\).
Weight \(=1 ; \quad d_{15}{ }^{-}=2, d_{23^{-}}=3, d_{31^{-}}=0, d_{31^{+}}=2, \quad d_{45^{-}}=0, d_{45^{+}}=0\).
Weight \(=1 ; \quad d_{51}{ }^{-}=0, d_{51}{ }^{+}=277832\).
```

Note that the results obtained comparatively mimicked the situational scenario, if not better. For weight $=5$, note that $d_{11}^{-}=0$ meaning that the capacity for first year intake of the Bioscience and Biotechnology is maximized such that $A_{1}=C_{1}=260$ compared to the current value of 257 . The variable $d_{24}=0$ means that the total capacity of students in the Mathematical Sciences is also maximized such that $D_{4}=T_{4}=650$ compared to the current value of 649. Affirmative action ratio in the Chemical Sciences and Food Technology is only overachieved by not more than one student as indicated by $d_{33}{ }^{-}=0$ and $d_{33}{ }^{+}=0.4$. The values $d_{41}{ }^{-}=5$ and $d_{41}{ }^{+}=0$ indicate that there is room of 5 more students in the Bioscience and Biotechnology to make up the 17 to 1 student-staff ratio. In fact the model provide a better affirmative action to that of the required ratio

$$
\mathrm{Q}_{1}=0.75, \mathrm{Q}_{2}=0.66, \mathrm{Q}_{3}=0.51, \mathrm{Q}_{4}=0.60 \text { and } \mathrm{Q}_{5}=0.59 .
$$

when the model provided ratios

$$
\mathrm{X}_{1} / \mathrm{A}_{1}=0.758, \mathrm{X}_{2} / \mathrm{A}_{2}=0.661, \mathrm{X}_{3} / \mathrm{A}_{3}=0.512, \mathrm{X}_{4} / \mathrm{A}_{4}=0.601 \text { and } \mathrm{X}_{5} / \mathrm{A}_{5}=0.591
$$

whereas current figures,

$$
\mathrm{X}_{1} / \mathrm{A}_{1}=0.755, \mathrm{X}_{2} / \mathrm{A}_{2}=0.663, \mathrm{X}_{3} / \mathrm{A}_{3}=0.512, \mathrm{X}_{4} / \mathrm{A}_{4}=0.604 \text { and } \mathrm{X}_{5} / \mathrm{A}_{5}=0.586
$$

This ratio affects the values of $\mathrm{X}_{i}$ and $\mathrm{Y}_{i}$ since the sum of these two variables equals $\mathrm{A}_{i}$. The extra sum of staff in the department with regard to the $L$ value allows the department to schedule study leave to its staff. Note that $L_{1}=43$ is equal to the real value. This signify the conformity to the highest weightage given to the Bioscience and Biotechnology, and a more flexible deviations to other centres. Note also the ratio

$$
D_{1} / L_{1}=17.12, D_{2} / L_{2}=18.12, D_{3} / L_{3}=16.82, D_{4} / L_{4}=15.12, \quad D_{5} / L_{5}=12.00
$$

and compare these values to the required student-staff ratio of

$$
R_{1}=17, R_{2}=18, R_{3}=17, R_{4}=15, R_{5}=12 .
$$

However, note that the value of the deviational variable $d_{12}=21$ is quite high even though the weight is second highest. This implies that the number of students admitted into the centre of Physical Sciences is twenty one students less than the capacity of that centre even though we placed the second highest weightage compared to the other centres. Thus we will seek to rectify this situation by placing a certain priority level to this particular deviation variable. In other words, weightage alone cannot emphasise the need to satisfy or optimize a particular constraint. Note that the aspiration level for budget expenditure of RM 2278490 is exceeded by RM 277832 as indicated by the variable $d_{51}{ }^{+}$. The Faculty of Science and Technology should seek an allocation of $J=$ RM 2556322 to run the faculty without the need to request a further RM 277832 at a later date. However, this figure is only RM 8048 more than the current budget.

## 4. CONCLUSION

We have successfully obtained the results of the weighted non-preemptive goal programming model. Based on the results obtained, we were able to undertake an indepth discussion on the deviation variables based on the given weights and relate the findings to the weights assigned to these variables. From the discussion of these deviational variables, we can verify that the results of the models conform to our requirement of fulfilling the goals in accordance to the corresponding weights of the five departments in the Faculty of Science and Technology. Thus we believe the model can be used for policy-making in the decision process of future allocation of students to academic departments.

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