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COMPLEX TRANSFORMATIONS TO SOLVE COSMOLOGICAL CONSTANT PROBLEM

Samy J. Litiloly
Mechanical Engineering Department, Faculty of Engineering – Pattimura University
Kampus Teknik UNPATTI, Jl. Ir. M. Putuhena, Poka, Ambon – Maluku, Indonesia 97233
E-mail : samyjunusl@yahoo.com

ABSTRACT

In this paper, a new symmetry argument in a vacuum state with strictly vanishing vacuum energy has been studied. This argument exploits the well-known feature that deSitter and Anti–deSitter space are related by analytic continuation in complex analysis. When we drop boundary and hermiticity conditions on quantum fields, we get as many negative as positive energy states, which are related by transformations to complex space. In this case, we have explored indirectly a new perspective to solve cosmological constant problem.

Keywords *: Analytic continuation, cosmological constant problem.*

INTRODUCTION

The constant of cosmological problem is the one of major obstacles for both particle physics and cosmology. The question is why is the effective cosmological constant, Λ_{eff} , defined as $\Lambda_{eff} = \Lambda + 8\pi G\langle\rho\rangle$ so closed to zero[1]. As is well known, different contributions to the vacuum energy density from particle physics would naively give a value for $\langle\rho\rangle$ of order M_P^4 , which then would have to be (nearly) cancelled by the unknown "bare" value of Λ.

This cancellation has to be precise to about 120 decimal places if we compare the zero-point energy of a scalar field, using the Planck scale as a cut-off, and experimental value of $\rho_{vac} = \langle\rho\rangle + \Lambda / 8\pi G$, being 10^{-47} GeV^4. As is well known, even if we take a TeV scale cut-off, the difference between experimental and theoretical results still requires a fine-tuning of about 50 orders of magnitude. This magnificent fine-tuning seems to suggest that we fail to observe something that is absolutely essential. In their lectures [1, 2], 't Hooft and Prokopec gave the different proposals that have occurred and we discuss in more detail a scenario that has been introduce in [3] and [4], based on symmetry with respect to a transformation towards imaginary values of the space-time coordinates: $x^\mu \rightarrow i\, x^\mu$. This symmetry entails of a new definition of vacuum state, as the unique state that is invariant under this transformation. Since curvature switches sign, this vacuum state must be associated with zero curvature, hence zero cosmological constant. The most striking and unusual feature of the symmetry is the fact that the *boundary conditions* of physical states are not invariant. Physical states obey boundary conditions when the real parts of the coordinates tend to infinity, not the imaginary parts. This is why all physical states, except the vacuum, must break the symmetry. We will argue that a vanishing cosmological constant could be a consequence of the specific boundary conditions of the vacuum, upon postulating this complex symmetry.

The fact that we are transforming real coordinates into imaginary coordinates implies, that hermitean operators are transformed into operators whose hermiticity properties are modified. Taking the hermitean conjugate of an operator requires knowledge of the boundary conditions of a state. The transition from x to ix requires that the boundary conditions of the states are modified. For instance, wave functions Φ that are periodic in real space, are now replaced by waves that are exponential expressions of x , thus periodic in ix. But we are forced to do more than that. Also the creation and annihilation operators will transform, and their commutator algebra in complex space is not priori clear; it requires careful study.

Thus, the symmetry that we are trying to identify is a symmetry of laws of nature *prior* to imposing any boundary conditions. Demanding invariance under $x_\mu \rightarrow x_\mu + a_\mu$ where a_μ may be real or imaginary, violates boundary conditions at $\Phi \rightarrow \infty$, leaving only one state invariant: the physical vacuum.

CASSICAL SCALAR FIELD

To set our notation, consider a real, classical, scalar field $\Phi(x)$ in D space-time dimensions, with Lagrangian

$$L = -\frac{1}{2}\left(\partial_\mu \Phi\right)^2 - V\left(\Phi(x)\right) \ , \qquad V(\Phi) = \frac{1}{2}m^2\Phi^2 + \lambda\Phi^4 \ .$$

(1)

Adopting the metric convention (- + + +), we write the energy-momentum tensor as

$$T_{\mu\nu}(x) = \partial_\mu \Phi(x)\ \partial_\nu \Phi(x) \ + \ g_{\mu\nu} L\left(\Phi(x)\right) \ . \tag{2}$$

The Hamiltonian H is

$$H = \int d^{D-1}\vec{x}\, T_{00}(x)\ ; \qquad T_{00} = \frac{1}{2}\Pi^2 + \frac{1}{2}\left(\vec{\partial}\Phi\right)^2 + V(\Phi)\ ; \qquad \Pi(x) = \partial_0 \Phi(x)\ . \tag{3}$$

Write our transformation as $x^\mu = iy^\mu$, after which all coordinates are rotated in their complex planes such that y^μ will become real. For redefined notions in y space, we use subscripts or superscripts y , e.g., $\partial_\mu^y = i\,\partial_\mu$. The field in y space obeys the Lagrange equations with

$$L_y \ = \ -L \ = -\frac{1}{2}\left(\partial_\mu^y \Phi\right)^2 \ + \ V(\Phi) \ \ ; \tag{4}$$

$$T_{\mu\nu}^y \ = \ -T_{\mu\nu} \ = \partial_\mu^y \Phi(iy)\partial_\nu^y \Phi(iy) + \ g_{\mu\nu} L_y\left(\Phi(iy)\right) \ . \tag{5}$$

The Hamiltonian in y-space is

$$H = -\left(i^{D-1}\right)H_y \qquad , \ H_y = \int d^{D-1} y\, T_{00}^y \ \ ; \tag{6}$$

$$T_{00}^y = \frac{1}{2}\Pi_y^2 \ + \ \frac{1}{2}\left(\vec{\partial}_y \Phi\right)^2 - V(\Phi) \qquad , \ \Pi_y(y) = i\Pi(iy) \ . \tag{7}$$

If we keep only the mass term in the potential, $V(\Phi)$ = ½ $m^2\Phi^2$, the field obeys the Klein-Gordon equation. In the real x-space, its solutions can be written as

$$\Phi(x,t) = \int d^{D-1}p\left(a(p)e^{i(px)} \ + \ a^*(p)e^{-i(px)}\right) \ , \tag{8}$$

$$\Pi(x,t) = \int d^{D-1}p\, p^0 \left(-ia(p)e^{i(px)} \ + \ i\,a^*(p)e^{-i(px)}\right) \ \ ; \tag{9}$$

$$p^0 = \sqrt{\ \vec{p}^2 \ + \ m^2} \quad , \quad (px) \equiv \vec{p}\cdot\vec{x} \ - p^0 t \ \ , \tag{10}$$

where $a(p)$ is just a c-number.

Analytically continuing these solutions to complex space, yields:

$$\Phi(iy, i\tau) \qquad = \int d^{D-1}q\left(a_y(q)e^{i(qy)} \ + \ \hat{a}_y(q)e^{-i(qy)}\right) \ , \tag{11}$$

$$\Pi_y(y,\tau) \ = \ i\Pi(iy,i\tau) \ = \int d^{D-1}q\ q^0\left(-ia_y(q)e^{i(qy)} \ + \ i\,\hat{a}_y(q)e^{-i(qy)}\right) \ \ ; \tag{12}$$

$$q^0 = \sqrt{\ \vec{q}^2 \ - \ m^2} \quad , \quad (qy) \equiv \vec{q}\cdot\vec{y} \ - q^0\tau \ \ . \tag{13}$$

The new coefficients could be analytic continuations of the old ones,

$$a_y = (-i)^{D-1}a(p) \ , \qquad \hat{a}_y(q) = (-i)^{D-1}a^*(q) \ , \qquad p^\mu = -iq^\mu \quad , \tag{14}$$

but this makes sense only if the $a(p)$ would not have singularities that we cross when shifting the integration contour. Note, that, since D = 4 is even, the hermiticity relation between $a_y(q)$ and $\hat{a}_y(q)$ is lost. We can now consider solutions where we restore them:

$$\hat{a}_y(q) = a_y^*(q) \quad , \tag{15}$$

while also demanding convergence of the q integration. Such solutions would not obey acceptable boundary conditions in x-space, and the fields would be imaginary rather than real, so these are unphysical solutions. The important property that we concentrate on now, however, is that, according to Eq. (5), these solutions would have to opposite for $T_{\mu\nu}$.

Of course, the field in y-space appears to be tachyonic, since m^2 is negative. In most of recent discussions, we should put m=0. A related transformation with the objective of $T_{\mu\nu} \rightarrow - T_{\mu\nu}$ was made by Kaplan and Sundurm in [5]. Non-Hermitian Hamiltonians were also studied by Bender at al. in for example [6, 7].

GRAVITY

Consider Einstein's equations:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \Lambda g_{\mu\nu} = -8\pi G T_{\mu\nu} \quad . \tag{16}$$

Writing

$$x^\mu = i\, y^\mu = i(y, \tau) \ , \qquad g^y_{\mu\nu}(y) \rightarrow g_{\mu\nu}(x = iy) \ , \tag{17}$$

and defining the Riemann tensor in y-space using the derivatives ∂^y_μ , we see that

$$R^y_{\mu\nu} = -R_{\mu\nu}(iy) \quad . \tag{18}$$

Clearly, in y-space, we have the equation

$$R^y_{\mu\nu} - \frac{1}{2} g^y_{\mu\nu} R^y + \Lambda g^y_{\mu\nu} = +8\pi G T_{\mu\nu}(iy) = -8\pi G T^y_{\mu\nu} \quad . \tag{19}$$

Thus, Einstein's equations are invariant except for the cosmological constant term.

A related suggestion was made in [8]. In fact, we could consider formulating the equations of nature in the full complex space z = x + iy , but then everything becomes complex. The above transformation is a one-to-one map from real space $\Re^3$ to the purely imaginary space $\Im^3$, where again real equations emerge.

The transformation from real to imaginary coordinates naturally relates deSitter space with anti-deSitter space, or, a vacuum solution with positive cosmological constant to a vacuum solution with negative cosmological constant. Only if the cosmological constant is zero, a solution can map into itself by such a transformation. None of the excited states can have this invariance, because they have to obey boundary conditions, either in real space, or in imaginary space.

SECOND QUANTIZATION

We now turn our attention to second-quantized particle theories, and we know that the vacuum state will be invariant, at least under all complex translations. Not only the hermiticity properties of field operators are modified in the transformation, but now also the commutation rules are affected. A scalar field $\Phi(x)$ and its conjugate, $\Pi(x)$, often equal to $\dot{\Phi}(x)$, normally obey the commutation rule

$$\left[\Pi(\vec{x}, t),\ \Phi(\vec{x}', t)\right] = -i\delta^3(\vec{x} - \vec{x}') \quad , \tag{20}$$

where the Dirac delta function $\delta(x)$ may be regarded as

$$\delta(x) = \sqrt{\frac{\lambda}{\pi} e^{-\lambda x^2}} \quad , \tag{21}$$

in the limit $\lambda \uparrow \infty$. If $\vec{x}$ is replaced by $i\vec{y}$, with $\vec{y}$ real, then the commutation rules are

$$\left[\Pi(i\vec{y}, t),\ \Phi(i\vec{y}', t)\right] = -i\delta^3\big(i(\vec{y} - \vec{y}')\big) \quad , \tag{22}$$

but, in (21) we see two things happen:

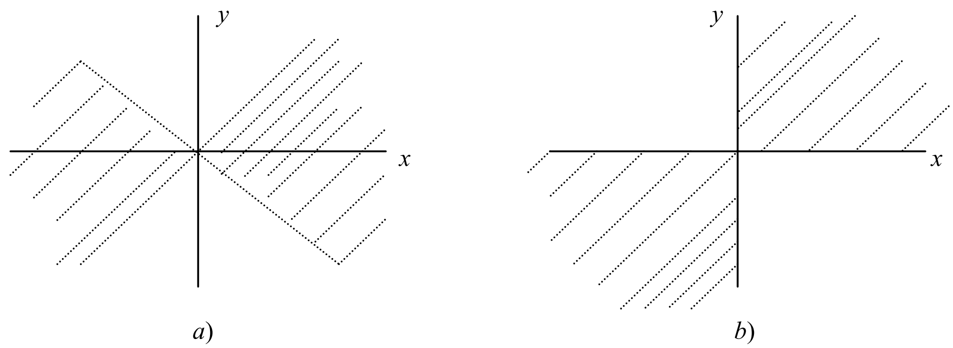
(i) This delta function does not go to zero unless its argument x lies in the right or left quadrant of Fig.1. Now, this can be cured if we add an imaginary part to λ , namely $\lambda \rightarrow -i\mu$, with μ real. Then the function (21) exists if $x = r\, e^{i\theta}$, with $0 < \theta < \frac{1}{2}\pi$.

But then,

(ii) If $x = iy$, the sign of μ is important. If $\mu > 0$, replacing $x = iy$, the delta function becomes

$$\delta(iy) = \sqrt{\frac{-i\mu}{\pi}}\; e^{-i\mu y^2} \;\; \rightarrow \;\; -i\delta(y) \quad , \tag{23}$$

which would be $+\,i\delta(y)$ had we chosen the other sign for μ .
We conclude that the sign of the square root in Eq. (21) is ambiguous.



Region in complex space where the Dirac delta function is well-defined,
(a) if λ is real, (*b*) if μ is real and positive.

There is another way to phrase this difficulty. The commutation rule (20) suggests that either the field $\Phi(\vec{x},t)$ or $\Pi(\vec{x},\,t)$ must be regarded as a distribution. Take the field Π. Consider test function $f(\vec{x})$, and write

$$\Pi(f,t) \equiv \int f(\vec{x})\,\Pi(\vec{x},t)\; d^3\vec{x} \quad ; \qquad \left[\Pi(f,t),\,\Phi(\vec{x},t)\right] = -i\,f(\vec{x}) \quad . \tag{24}$$

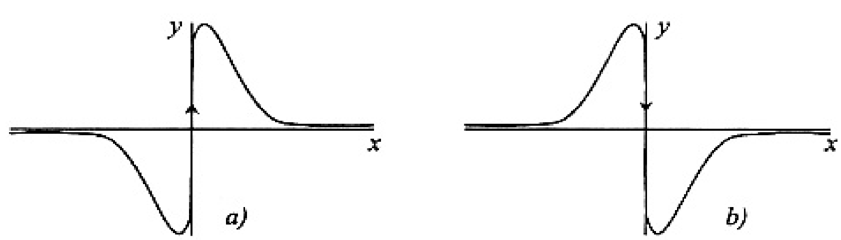
As long as $\vec{x}$ is real, the integration contour in Eq. (24) is well defined. If, however, we choose x = iy , the contour must be taken to be in the complex plane, and if we only wish to consider real y, then the contour must be along the imaginary axis. This would be allowed if $\Pi(\vec{x},y)$ is holomorphic for complex $\vec{x}$, and the end points of the integration contour should not be modified.

For simplicity, let us take space to be one-dimensional. Assume that the contour becomes as in Fig. 2a. In the y space, we have

$$\Pi(f,t) \equiv \int_{-\infty}^{\infty} f(iy)\,\Pi(iy)\,d(iy) \quad ; \qquad \left[\Pi(f,t),\,\Phi(iy,t)\right] = -if(iy) \quad , \tag{25}$$

so that

$$\left[\Pi(iy,t),\,\Phi(iy',t)\right] = -\,\delta(y-y')\;. \tag{26}$$



Integration contour for the commutator algebra (24), (*a*) and (*b*) being two distinct choices.

Note now that we could have chosen the contour of Fig. 2b instead. In that case, the integration goes in the opposite direction, and the commutator algebra in Eq. (26) receives the opposite sign. Note also that, if we would be tempted to stick to one rule only, the commutator algebra would receive an overall minus sign if we apply the transformation x → iy twice.

The general philoshopy is now that, with these new commutation relations in y-space, we could impose conventional hermiticity properties in y-space, and then consider states as representations of these operators. How do individual states then transform from x-space to y-space or *vice versa* ? We expect to obtain non-normalizable states, but the situation is worse than that. Let us again consider one space-dimension, an begin with defining the annihilation and creation operators *a(p)* and a^{+}*(p)* in x-space:

$$\Phi(x,t) = \int \frac{dp}{\sqrt{2\pi \cdot 2p^0}} \left(a(p)\, e^{i(px)} + a^+(p)\, e^{-i(px)} \right) , \tag{27}$$

$$\Pi(x,t) = \int \frac{dp\sqrt{p^0}}{\sqrt{2 \cdot 2\pi}} \left(-ia(p)\, e^{i(px)} + ia^+(p)\, e^{-i(px)} \right) \tag{28}$$

$$p^0 = \sqrt{\vec{p}^{\,2} + m^2} \quad , \quad (px) \equiv \vec{p} \cdot \vec{x} - p^0\, t \tag{29}$$

$$a(p) = \int \frac{dx}{\sqrt{2\pi \cdot 2p^0}} \left(p^0\, \Phi(x,t) + i\Pi(x,t) \right) e^{-i(px)} \quad , \tag{30}$$

$$a^+(p) = \int \frac{dx}{\sqrt{2\pi \cdot 2p^0}} \left(p^0\, \Phi(x,t) - i\Pi(x,t) \right) e^{i(px)} \quad . \tag{31}$$

Insisting that the commutation rules $[\, a(p),\, a^+(p')\,] = \delta\,(p - p')$ should also be seen in y-space operators:

$$[\, a_y(q),\, \hat{a}_y(q')\,] = \delta\,(q - q') , \tag{32}$$

we write, assuming $p^0 = -i\, q^0$ and $\Pi = -i \dfrac{\partial \Phi}{\partial \tau}$ for free fields,

$$\Phi(iy, i\tau) = \int \frac{dq}{\sqrt{2\pi \cdot 2q^0}} \left(a_y(q)\, e^{i(qy)} + \hat{a}_y(q)\, e^{-i(qy)} \right) , \tag{33}$$

$$\Pi(iy, i\tau) = \int \frac{dq\sqrt{q^0}}{\sqrt{2 \cdot 2\pi}} \left(-\, a_y(q)\, e^{i(qy)} + \hat{a}_y(q)\, e^{-i(qy)} \right) \tag{34}$$

$$q^0 = \sqrt{\vec{q}^{\,2} - m^2} \quad , \quad (qy) \equiv \vec{q} \cdot \vec{y} - q^0\, \tau \tag{35}$$

$$a_y(q) = \int \frac{dy}{\sqrt{2\pi \cdot 2q^0}} \left(q^0\, \Phi(iy, i\tau) - \Pi(iy, i\tau) \right) e^{i(qy)} \quad , \tag{36}$$

$$\hat{a}_y(q) = \int \frac{dy}{\sqrt{2\pi \cdot 2q^0}} \left(q^0\, \Phi(iy, i\tau) + \Pi(iy, i\tau) \right) e^{-i(qy)} \quad , \tag{37}$$

so that the commutator (32) agrees with the field commutators (26). In most of my considerations, we will have to take m = 0 ; we leave m in our expressions just to show its sign switch.

In x-space, the the fields Φ and π are real, and the exponents in Eq. (33) – (37) are all real, so the hermiticity relations are $a_y^+ = a_y$ and $\hat{a}_y^+ = \hat{a}_y$. Now, we replace this by

$$\hat{a}_y = a_y^+ \ . \tag{38}$$

The Hamiltonian for a free field reads

$$H = i \int_{-\infty}^{\infty} dy \left(\frac{1}{2} \Pi(iy)^2 - \frac{1}{2} \left(\partial_y \Phi(iy) \right)^2 + \frac{1}{2} m^2 \Phi(iy)^2 \right)$$

$$= -i \int dq\ q^0 \left(\hat{a}_y(q)\, a_y(q) + \frac{1}{2} \right) = -i \int dq\ q^0\, (n + \frac{1}{2}) \quad . \tag{39}$$

Clearly, with the hermiticity condition (38), the Hamiltonian became purely imaginary, as in Section 2. Also, the zero point fluctuations still seem to be there. However, we have not yet addressed the operator ordering. Let us take a closer look at the way individual creation and annihilation operators transform. We now need to set m = 0 , p^0 = |p| , q^0 = |q| . In order to compare the creation and annihilation operators in real space-time with those in imaginary space time, substitute Eqs. (33) and (34) into (30), and the converse, to obtain

$$a(p) = \iint \frac{dx\,dq}{2\pi\sqrt{4p^0q^0}}\left\{(p^0 - i\,q^0)\,a_y(q)e^{(q-ip)x} + (p^0 + i\,q^0)\,\hat{a}_y(q)e^{(-q-ip)x}\right\}, \quad (40)$$

$$a_y(q) = \iint \frac{dy\,dp}{2\pi\sqrt{4p^0q^0}}\left\{(q^0 + i\,p^0)\,a(p)e^{(-iq-p)y} + (q^0 - i\,p^0)\,a^+(p)e^{(-iq+p)y}\right\}. \quad (41)$$

The difficulty with these expressions is the fact that the x- and the y- integrals diverge. The following procedure is proposed. Let us limit ourselves to the case that, in Eqs. (36) and (37), the y-integration is over a finite box only: $|y| < L$, in which case $a_y(q)\sqrt{2q^0}$ will be an entire analytic function of q. Then, in Eq. (40), we can first shift the integration contour in complex q-space by amount ip up or down, and subsequently rotate the x-integration contour to obtain convergence. Now the square roots occurring explicitly in Eqs. (40) and (41) are merely the consequence of a choice of normalization, and could be avoided, but the root in the definition of p^0 and q^0 are much more problematic. In principle we could take any of the two branches of the roots. However, in our transformation procedure we actually *choose* $q^0 = -\,i\,p^0$ and the second parts of Eqs. (40) and (41) simply cancel out. Note that, had we taken the other sign, i.e. $q^0 = i\,p^0$, this would have affected the expression for $\Phi(iy, i\tau)$ such, that we would still end up with the same final result. In general, the x-integration yields a delta function constraining q to be $\pm\, ip$, but q^0 is chosen to be on the branch $-\,i\,p^0$, in both terms of this equation (q^0 normally does not change sign if q does). Thus, we get, from Eqs. (40) and (41), respectively,

$$a(p) = i^{1/2}a_y(q) \;, \qquad q = i\,p \;, \qquad q^0 = i\,p^0 \;, \quad (42)$$

$$a_y(q) = i^{-1/2}a(p) \;, \qquad p = -\,i\,q \;, \qquad p^0 = -\,i\,q^0 \;, \quad (43)$$

so that $a(p)$ and $a_y(q)$ are analytic continuations of one another. Similarly,

$$a^+(p) = i^{1/2}\,\hat{a}_y(q) \;, \quad \hat{a}_y(q) = i^{-1/2}a^+(p) \;, \quad p = -\,i\,q \;, \quad p^0 = -\,i\,q^0 \;. \quad (44)$$

There is no Bogolyubov mixing between a and a^+. Note that these expressions agree with the transformation law of the Hamiltonian (39).

Now that we have a precisely defined transformation law for the creation and annihilation operators, we can find out how the states transform. The vacuum state $|0\rangle$ is defined to be the state upon which all annihilation operators vanish. We now see that this state is invariant under all our transformations. Indeed, because there is no Bogolyubov mixing, all N particle states transform into N particle states, with N being invariant. The vacuum is invariant because 1) creation operators transform into creation operators, and annihilation operators into annihilation operators, and because 2) the vacuum is translation invariant.

The Hamiltonian is transformed into $-\,i$ times Hamiltonian (in the case $D = 2$); the energy density T_{00} into $-T_{00}$, and since the vacuum is the only state that is invariant, it must have $T_{00} = 0$ and it must be the only state with this property.

MAXWELL FIELDS

This can now easily to include the Maxwell action as well. In flat space-time:

$$S = -\int d^3x\,\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) \;, \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \;. \quad (45)$$

The action is invariant under gauge transformations of the form

$$A_\mu(x) \;\rightarrow\; A_\mu(x) + \partial_\mu\xi(x) \;. \quad (46)$$

Making use of this freedom, we impose the Lorentz condition $\partial_\mu A_\mu = 0$, such that the equation of motion $\partial_\mu F^{\mu\nu} = 0$ becomes $A^\mu = 0$. As is well known, this does not completely fix the gauge, since transformations like (46) are still possib□ provided $\xi = 0$. This remaining gauge freedom can be used to set $\nabla\cdot\vec{A} = 0$, denoted Coulomb gauge, which sacrifices manifest Lortenz invariance. The commutation relations are

$$[E^i(x,t),\ A_j(x',t)] = i\delta_{ij}^{tr}(\vec{x} - \vec{x}') \quad , \tag{47}$$

where

$$E^k = \frac{\partial L}{\partial \dot{A}_k} = -\dot{A}^k - \frac{\partial A_0}{\partial x^k} \quad , \tag{48}$$

is the momentum conjugate to A^k , which we previously called Π , but it is here just a component of the electric field. The transverse delta-function is defined as

$$\delta_{ij}^{tr}(\vec{x} - \vec{x}') \equiv \int \frac{d^3 p}{(2\pi)^3} e^{i\,\vec{p}(\vec{x}-\vec{x}')} \left(\delta_{ij} - \frac{p_i\, p_j}{\vec{p}^{\,2}} \right) \quad , \tag{49}$$

such that its divergence vanishes. In Coulomb gauge, $\vec{A}$ satisfies the wave equation $\Box \vec{A} = 0$, and we write

$$\vec{A}(x,t) = \int \frac{d^3 p}{(2\pi)^3 \sqrt{2p^0}} \sum_{\lambda=1}^{2} \vec{\varepsilon}(p,\lambda)\left(a(p,\lambda)\, e^{i(px)} + a^+(p,\lambda)\, e^{-i(px)} \right) \quad , \tag{50}$$

where $\vec{\varepsilon}(p,\lambda)$ is the polarization vector of the gauge field, which satisfies $\vec{\varepsilon} \cdot \vec{p} = 0$ from the Coulomb condition $\nabla \cdot \vec{A} = 0$. Moreover, the polarization vectors can be chosen to be orthogonal $\vec{\varepsilon}(p,\lambda) \cdot \vec{\varepsilon}(p,\lambda') = \delta_{\lambda\lambda'}$ and satisfy a completeness relation

$$\sum_{\lambda} \varepsilon_m(p,\lambda)\, \varepsilon_n(p,\lambda) = \left(\delta_{mn} - \frac{p_m\, p_n}{\vec{p}^{\,2}} \right) \quad . \tag{51}$$

The commutator relation between the creation and annihilation operators becomes

$$[\, a(p,\lambda),\ a^+(p',\lambda)\,] = \delta(\vec{p} - \vec{p}')\, \delta_{\lambda\lambda'} \quad , \tag{52}$$

in which the polarization vectors cancel out due to their completeness relation.

In complex space, the field A_μ thus transforms analogously to the scalar field, with the only addition that the polarization vectors $\vec{\varepsilon}_\mu(p)$ will now become function of complex momentum $\vec{q}$. However, since they do not satisfy a particular algebra, like the creation and annihilation operators, they do not cause any additional difficulties. The commutation relations between the creation and annihilation operators behave similarly as in the scalar field case, since the second term in the transverse delta-function (49), and the polarization vector completeness relation (51), is invariant when transforming to complex momentum.

Thus we find

$$F_{\mu\nu}(x,t)\, F^{\mu\nu}(x,t) \ \rightarrow\ -F_{\mu\nu}(iy,i\tau)\, F^{\mu\nu}(iy,i\tau) \quad , \tag{53}$$

and again T_{00} flips sign, as the energy momentum tensor reads:

$$T_{\mu\nu} = -F_{\mu\alpha} F_{\nu}^{\alpha} + \frac{1}{4} F_{\alpha\beta} F^{\alpha\beta} \eta_{\mu\nu} \quad . \tag{54}$$

In term of the E and B fields, which are given by derivatives of A_μ , $E_i = F_{0i}$, $B_k = \frac{1}{2}\varepsilon_{ijk} F_{jk}$, we have:

$$T_{00} = \frac{1}{2}\left(E^2 + B^2 \right) \ \rightarrow\ -T_{00} \quad , \tag{55}$$

which indicates that the electric and magnetic fields become imaginary. A source term $J^\mu A_\mu$ can also be added to the action (45), if one imposes that $J^\mu \rightarrow -J^\mu$, in which case the Maxwell equations $\partial_\mu F^{\mu\nu} = J^\nu$ are covariant.

Implementing gauge invariance in imaginary space is also straightforward. The Maxwell action and Maxwell equations are invariant under $A_\mu(x,t) \to A_\mu(x,t) + \partial_\mu \xi(x,t)$. In complex space time this becomes

$$A_\mu(iy,i\tau) \;\to\; A_\mu(iy,i\tau) - i\partial_\mu(y,\tau)\xi(iy,i\tau) \tag{56}$$

and the Lorentz condition

$$\partial_\mu(x,t)A^\mu(x,t) = 0 \qquad \to \qquad i\partial_\mu(y,\tau)A^\mu(iy,i\tau) \tag{57}$$

In Coulomb gauge the polarization vectors satisfy

$$\vec{\xi}(q)\cdot\vec{q} \;=\; 0 \quad , \tag{58}$$

where q is imaginary momentum.

RELATION WITH BOUNDARY CONDITIONS

All particle states depend on boundary conditions, usually imposed on the real axis. One could therefore try to simply view the $x \rightarrow ix$ symmetry as a one-to-one mapping of states with boundary conditions imposed on $\pm x \rightarrow \infty$ to states with boundary conditions imposed on imaginary axis $\pm ix \rightarrow \infty$. At first sight, this mapping transforms positive energy particle states into negative energy particle states. The vacuum, not having to obey boundary conditions would necessarily have zero energy. However, this turns out not to be sufficient.

Solutions to Klein-Gordon equation, with boundary conditions imposed on imaginary coordinates are of the form:

$$\Phi_{im}(x,t) = \int \frac{dp}{\sqrt{2\pi\cdot 2p^0}}\left(a(p)e^{(px)} + \hat{a}(p)e^{-(px)}\right) , \qquad p^0 = \sqrt{p^2 + m^2} \quad , \tag{59}$$

written with the supscript " im " to remind us that this is the solution with boundary conditions on the imaginary axis. With these boundary conditions, the field explodes for real valued $x \rightarrow \pm\infty$, whereas for the usual boundary conditions, imposed on the real axis, the field explodes for $ix \rightarrow \pm\infty$. Note that for non-trivial a and $\hat{a}$, this field now has a non-zero complex part on the real axis, if one insists that the second term is the Hermitian conjugate of the first, as is usually the case. This is a necessary consequence of this set up. However , we insist on writing $\hat{a} = a^+$ and returning to three spatial dimensions, we write for $\Phi_{im}(x,t)$ and $\Pi_{im}(x,t)$:

$$\Phi_{im}(\vec{x},t) \;=\; \int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2p^0}}\left(a_p\, e^{(px)} + a_p^+\, e^{-(px)}\right) ,$$

$$\dot{\Phi}_{im}(\vec{x},t) \;=\; \Pi_{im}(\vec{x},t) \;=\; \int \frac{d^3p}{(2\pi)^3}(-)\sqrt{\frac{p^0}{2}}\left(a_p\, e^{(px)} - a_p^+\, e^{-(px)}\right) ,$$

$$p^0 = \sqrt{\vec{p}^2 + m^2} \quad , \qquad (px) \equiv \vec{p}\cdot\vec{x} - p^0 t \quad , \tag{60}$$

and impose the normal commutation relations between a and a^+ :

$$[a_p, a^+_{p'}] = (2\pi)^3\,\delta^{(3)}(\vec{p}-\vec{p}\,') \quad . \tag{61}$$

Using Eq. (61), the commutator between Φ_{im} and Π_{im} at equal times, becomes:

$$[\Phi_{im}(\vec{x}), \Pi_{im}(\vec{x})] \;=\; \delta^{(3)}(\vec{x}-\vec{x}\,') \quad , \tag{62}$$

which differs by a factor of i from the usual relation, and by a minus sign, compared to Eq. (26). The energy-momentum tensor is given by

$$\left(T_{\mu\nu}\right)_{im} \;=\; \partial_\mu\Phi_{im}\,\partial_\nu\Phi_{im} \;-\; \frac{1}{2}\eta_{\mu\nu}\partial^k\Phi_{im}\partial_k\Phi_{im} \quad , \tag{63}$$

and thus indeed changes sign, as long as one considers only those contributions to a Hamiltonian that contain products of a and a^+ :

$$H^{diag}_{im} \;=\; \int \frac{d^3p}{(2\pi)^3}\,p^0\left(-a_p^+\,a_p - \frac{1}{2}[a_p, a_p^+]\right) \;=\; -H \quad . \tag{64}$$

However, the remaining parts give a contribution that is rapidly diverging on the imaginary axis

$$\left(T_{\mu\nu}^{non-diag}\right)_{im} = a^2\, e^{2(px)} + (a^+)^2\, e^{-2(px)} \quad , \tag{65}$$

but which blows up for $\pm x \rightarrow \infty$. Note that when calculating vacuum expectation values, these terms give no contribution.

The summarize, one can only construct such a symmetry, changing boundary conditions from real to imaginary coordinates, in a very small box. This was to be expected, since we are comparing hyperbolic functions with their ordinary counterparts, $\sinh(x)$ vs. $\sin(x)$, and they are only identical functions in a small neighborhood around the origin.

CONCLUSIONS

It is natural to ascribe the extremely tiny value of the cosmological constant to some symmetry. Until now, the only symmetry that showed promises in this respect has been supersymmetry. It is difficult, however, to understand how it can be that supersymmetry is obviously strongly broken by all matter representations, whereas nevertheless the vacuum state should respect it completely. This symmetry requires the vacuum fluctuations of bosonic fields to cancel very precisely against those of the fermionic field, and it is hard to see how this can happen when fermionic and bosonic fields have such dissimilar spectra.

The symmetry proposed in this paper is different. It is suspected that the field equations themselves have a larger symmetry than the boundary conditions for the solutions. It is the boundary conditions, and the hermiticity conditions on the fileds, that force all physical states to have positive energies. If we drop these conditions, we get as many negative energy as positive energy states, and indeed, there may be a symmetry relating positive energy with negative energy. This is the most promising beginning of an argument why the vacuum state must have strictly vanishing gravitational energy.

The fact that the symmetry must relate real to imaginary coordinates is suggested by the fact that deSitter and Anti-deSitter space are related by analytic continuation, and that their cosmological constant have opposite sign.

Unfortunately, it is hard to see how this kind of symmetry could be realized in the known interaction types seen in the sub-atomic particles. At first sight, all mass term are forbidden. However, we could observe that all masses in the Standard Model are due to interactions, and it could be that fields with positive mass squared are related to tachyonic fields by our symmetry. The one scalar field in the Standard Model is the Higgs field. Its self interaction is described by a potential $V_1(\Phi) = \frac{1}{2}\lambda(\Phi^+\Phi - F^2)$, and it is strongly suspected that parameter λ is unnaturally small. Our symmetry would relate it to another scalar field with opposite potential: $V_2(\Phi_2) = -V_1(\Phi_2)$. Such a field would have no vacuum expectation value, and, according to perturbation theory, a mass that is the Higgs mass divided by $\sqrt{2}$. Although explicit predictions would be premature, this does suggest that a theory of this kind could make testable predictions, and it is worth-while to search for scalar fields that do not contribute to the Higgs mechanism at LHC, having a mass somewhat smaller than the Higgs mass. We are hesitant with this particular prediction because the negative sign in its self interaction potential could lead to unlikely instabilities, to be taken care of by non-perturbative radiative corrections.

The symmetry we studied in this paper would set the vacuum energy to zero and has therefore the potential to explain a vanishing cosmological constant. In light of the recent discoveries that the universe appears to be accelerating [9], one could consider a slight breaking of this symmetry. This is a non-trivial task that we will have to postpone to further study. Note however, that our proposal would only nullify exact vacuum energy with equation of state w = -1. Explaining the acceleration of the universe with some dark energy component other than a cosmological constant, quintessence for example, therefore is not ruled out within this framework.

REFERENCES

[1] G. 't Hooft, 2009, *The Implications of Black Holes for Particle Physics*, The First USTP Lunch Talk, ITF and Spinoza Institute, Utrecht University, The Netherlands.

[2] T. Prokopec, 2009, *Cosmic Inflation*, The Third USTP Lunch Talk, ITF and Spinoza Institute, Utrecht University, The Netherlands.

[3] R. V. Churchill and J. W. Brown, 1990, *Complex Variables and Application*, 5th-Edition, Mc Graw Hill, New York, USA.
[4] S. J. Litiloly, 2009, *Mathematical Physics 2*, Notes on Lecture, FMIPA Physics Department, Pattimura University, Indonesia.
[5] D.E. Kaplan and R. Sundurm, 2006, *A Symmetry for the Cosmological Constant*, hep-th/0505265, ITF and Spinoza Institute, Utrecht University, The Netherlands.
[6] C.M. Bender, 2005, *Non-Hermitian Quantum Field Theory*, Int. J. Mod. Phys. A20, 4646 – 4652.
[7] C.M. Bender, H.F. Jones, and R.J. Rivers, 2005, *Dual PT-symmetric Quantum Field Theories*, Phys. Lett. B62, 333 – 340.
[8] R. Erdem, *A symmetry for Vanishing Cosmological Constant in an Extra Dimensional Toy Model,* Phys. Lett. B621, 11 – 17.
[9] Private communications with ITF (Institute for Theoretical Physics) and Spinoza Institute, 2009 --- ,Utrecht University, The Netherlands
Taylor, W.A. (1991), *Optimization and Variation Reduction in Quality*, McGraw-Hill, Inc